

**Probability Theory I**  
**Basics of Probability Theory; Law of Large Numbers, Central  
Limit Theorem and Large Deviation**

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July 24, 2021

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